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ZZH coupling : A probe to the origin of EWSB ?

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Abstract

We argue that the ZZH coupling constitutes a simple probe of the nature of the scalar sector responsible for electroweak symmetry breaking. We demonstrate the efficacy of this measure through an analysis of four-dimensional models containing scalars in arbitrary representation of $SU(2) \times U(1)$, as well as extra-dimensional models with a non-factorizable geometry. A possible role for the $t\bar{t}H$ couplings is also discussed.

1 Introduction

Despite its tremendous success, the Standard Model (SM) still lacks any direct experimental information regarding the electroweak symmetry breaking (EWSB) sector. While the Higgs mechanism is a very plausible candidate, giving masses, as it does, to both the weak gauge bosons as well as the fermions, it predicts one or more physical spin-0 states, none of which has been found so far. Detection of these scalars (pseudo-scalars) is one of the major goals of the particle physics experiments being built at the moment.

This very lack of experimental information precludes any definitive statement regarding the Higgs sector. While the minimal choice of a $SU(2)$ doublet, as in the SM, is an aesthetically pleasing one, there is very little support for such a choice in terms of either phenomenological evidence or any theoretical understanding. Indeed, many alternatives to the SM have an extended Higgs sector. In particular, supersymmetric theories, proposed to cure the bad high energy behaviour of the Higgs mass in SM, necessarily have at least two scalar doublets taking part in the EWSB. Somewhat similar is the situation with grand unified theories (GUT). Apart from containing typically more than one Higgs responsible for EWSB, such scenarios often include extra low-mass scalars that do not directly take part in EWSB, but mix with the Higgs bosons thereby affecting their properties. Since search strategies for the Higgs(es) depend crucially on the possible decay channels, any such mixing will have very definite consequences. Examples are afforded by the recently proposed models for TeV scale gravity. The low-energy effective theory obtained upon compactification of the extra space time dimensions typically contain one or more scalars that do not contribute to the EWSB, but may mix with the SM Higgs, thus modifying the Higgs decay patterns in general.

In future experiments, we may see a number of neutral scalars. This would immediately imply that the Higgs sector is not the minimal one as chosen in the SM. On the other hand, nature may choose to have an extended scalar sector, but by a conspiracy of circumstances, experiments may find only one scalar. The important question is finding out what a detected scalar, or a set of scalars, can tell us about the mechanism of symmetry breaking [1, 2]. Effects of an extended Higgs sector were also the subject of discussion in refs.[3, 2, 1]. In ref.[2], for example, the focus was on differentiating between various supersymmetry breaking scenarios, if the Higgs mass were determined to be around 115 GeV, the current lower limit for the SM Higgs [4]. The authors of ref.[1], on the other hand, studied the HZZ coupling in some detail keeping in mind various scenarios for EWSB.

It, obviously, would be useful to construct a simple observable that would be sensitive to the nature of the EWSB mechanism. To this end, we concentrate on the coupling of the scalar(s) to the Z-bosons. Unlike the fermion couplings to the Higgs, this measure (alongwith the HWW couplings) receives contributions from any non-singlet scalar vacuum expectation value (VEV). Moreover, the experimental measurement of such couplings is of particular importance as these drive Higgs productions through of the Bjorken process [5] and gauge boson fusion [6].

While we have seemingly dismissed the fermion couplings to the physical scalars, they deserve a closer examination, particularly the ones involving the third generation. At the LHC, for example, the top Yukawa coupling would be measured with an accuracy $\mathcal{O}(20\%)$ for Higgs lighter than about 130 GeV [7]. At a linear collider operating at $\sqrt{s} = 750$ GeV and with an integrated luminosity 50 fb^{-1}, the same coupling is likely to be measured with an accuracy of $\mathcal{O}(10\%)$ for $m_H < 240$ GeV [8]. However, even an observed deviation from the SM value would not be enough to distinguish a scenario with more than one doublet Higgs from an exotic one. Some indirect inferences may be drawn though. For example, if the measured value of the top-Yukawa at the EW scale turns out to be significantly larger than the SM value, then, under renormalization group evolution, it would approach a non-perturbative regime much sooner than within the SM. This would be indicative of some new physics operative at that scale. Such an eventuality is a real possibility in scenarios wherein more than one scalar takes part in the EWSB. As the scalar(s) giving mass to top quark must then have a VEV (VEVs) smaller than the SM Higgs VEV, the top-Yukawa must be enhanced accordingly. Thus the structure of scalar sector and top Yukawa are highly correlated.

The plan of the rest of the article is as follows. In section 2, we will discuss the coupling of the Higgs to a pair of Z-bosons in extensions of SM wherein the scalar sector consists of, apart from the usual doublet of $SU(2)_L$, fields transforming in higher-dimensional representations. Also discussed, in this context, are constraints emanating from perturbativity of the top quark-Higgs coupling as also from the naturalness of cancellations in the ρ-parameter. In Section 3, we concentrate on the models of TeV scale gravity, especially those incorporating a non-factorisable geometry. We analyse the mixing of the graviscalars with the SM Higgs and the consequent modification of the couplings to the Z. Also studied is supersymmetric variation of such models. Finally we conclude in section 4.

2 ZZH coupling in four dimensional models

On account of its excellent agreement with almost all of current experimental data, the Standard Model serves as the natural template for any investigation of the Higgs sector. Within the SM, the ZZH coupling follows from the kinetic term $(D_\mu\phi)^\dagger D^\mu\phi$, where ϕ is the Higgs doublet and D_μ denotes the gauge covariant derivative. Clearly, if the Higgs sector were to be extended, one needs consider a sum of such individual kinetic terms, with the covariant derivative defined appropriately for each type of field.

There are good reasons for studying models with several scalars. Not only do such scalars occur in almost any extension of the Standard Model, even their representations may vary considerably. Perhaps the simplest such extension is afforded by models for spontaneous CP violation [9]. Then, there are the models for neutrino masses (and lepton number violation) which incorporate $SU(2)_L$ triplets, of both the real and the complex kind [10]. A further set of models are those that lead to gauge unification at high energy scales, either directly into a single gauge group or via some intermediate steps. Such scenarios, perforce, have to have additional Higgses that break the large symmetry down to the SM gauge group. The details, of course, depend on the choice of the initial gauge symmetry as well as the chain of symmetry breaking. As an example, let us consider the simplest model for gauge unification, namely $SU(5)$. The SM fermions are then contained in two disjoint representations, viz. the $\bar{\mathbf{5}}$ and $\mathbf{10}$. Since

$$\bar{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \mathbf{45}, \quad \mathbf{10} \times \mathbf{10} = \bar{\mathbf{5}} + \mathbf{45} + \mathbf{50} \;\text{ and }\; \bar{\mathbf{5}} \times \bar{\mathbf{5}} = \overline{\mathbf{10}} + \overline{\mathbf{15}}, \tag{1}$$

one can have gauge invariant *renormalizable* Yukawa couplings for scalars transforming as one of $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{45}$, and $\mathbf{50}$. Of these, only $\bar{\mathbf{5}}, \overline{\mathbf{45}}$ and $\overline{\mathbf{15}}$ contain neutral scalars — the first two in the doublet of the embedded $SU(2)_L$ while the last as a $Y = 1$ triplet — and may generate fermion or gauge boson masses. Apart from the abovementioned set of fields, one must also consider a scalar in the adjoint representation ($\mathbf{24}$) as that is the one responsible for breaking $SU(5)$ down to the Standard Model gauge group.

A particular problem that a generic GUT model faces is that of generating the correct low-energy mass spectrum. To facilitate this, in a natural manner, it is often necessary to consider Higgses in other representations. While, at the tree level, such scalars would couple to only the gauge bosons and the other (lower-dimensional) scalars in the theory:

$$\bar{\mathbf{5}} \times \mathbf{45} = \mathbf{24} + \ldots, \quad \overline{\mathbf{15}} \times \mathbf{15} = \mathbf{70}' + \ldots, \quad \overline{\mathbf{45}} \times \mathbf{45} = \mathbf{35} + \ldots \;, \tag{2}$$

these may also participate in non-renormalizable couplings involving fermions. It can be easily ascertained that these new scalars appearing in eq.(2) contain, in addition to the SM gauge group singlet, components transforming as one of triplets (hypercharge $Y = 0, 1$), 4-plets ($Y = 3/2, 1/2$) or 5-plets ($Y = 2, 1, 0$) of $SU(2)_L \otimes U(1)_Y$ [1].

A third set of examples, where more than one physical scalar emerges, is afforded by the supersymmetric models. If the Standard Model (SM) description were to be valid up to high energy scales, it would face the theoretical problem of the Higgs boson mass receiving potentially large radiative corrections. To counter this in nonsupersymmetric GUTs, one has to precisely fine

[1] The intermediate mass scalars, which couple to the gauge bosons, will change the running of the gauge couplings. E.g. the effect of new representations on $\sin^2\theta_W$ has been considered in [11]. The measured value of the weak mixing angle restricts the mass scales and representations of the new scalars.

tune between the bare mass and radiative corrections. Supersymmetry, on the other hand, offers a very natural solution to the problem, and without coming into conflict with any experimental observation. A major difference between the Standard Model and supersymmetric models is that the Higgs sector is necessarily extended to include at least two Higgs doublets instead of just the one needed in the Standard Model.

Having argued that it is natural for extensions of the Standard Model to contain more than one Higgs with a neutral component, let us now consider the Higgs couplings with the Z. Summing over all possible neutral Higgses, we have

$$L_{Z-H} = (g^2 + g'^2) Z^\mu Z_\mu \sum_i \left[\frac{1}{2} T_{3i}^2 H_i^0 H_i^0 + T_{3i}^2 v_i H_i^0 + \frac{1}{2} T_{3i}^2 v_i^2 \right] , \tag{3}$$

where T_i's are the isospins of the Higgs multiplets and v_i's are the VEVs of the corresponding neutral components. Since the last term in (3) gives the Z mass,

$$m_Z^2 = (g^2 + g'^2) \sum_i T_{3i}^2 v_i^2 , \tag{4}$$

it is obvious that, in the presence of many Higgses with nonzero VEVs, the magnitude of an individual VEV must be smaller than that of the SM one. Similar conclusions follow from the mass of the W-boson as well. In fact, one of the strongest constraints on VEVs of non-standard Higgses arises from the precision measurement of the electroweak ρ parameter. Defined as the ratio of the strengths of the charged– and the neutral currents, we have, at the tree level,

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} ,$$

with a Standard Model value of unity. While quantum corrections to the ρ-parameter have been calculated, for our analysis it suffices to consider only the tree level expression. In the presence of many Higgses, we have

$$\rho = \frac{\sum_i r_i \left[T_i(T_i+1) - T_{3i}^2 \right] v_i^2}{\sum_i 2 T_{3i}^2 v_i^2} . \tag{5}$$

with $r_i = 1/2$ for real representations and 1 otherwise. It is immediately apparent that the experimental observation of $\rho \approx 1$ can be naturally satisfied only for certain very specific choices of the Higgs representations. For all others, a degree of fine-tuning between the VEVs is necessary. Such a fine-tuning may be motivated though from other considerations [10]. Throughout the rest of the paper, we will assume that the ρ-parameter should be unity at the tree-level, thereby defining a constraint in the space of Higgs VEVs.

We now revert back to the coupling of the Z with the generic CP-even neutral Higgs H_i^0. These may be conveniently parametrized by

$$L_{H_i ZZ} = c_i \, \frac{g m_Z}{\cos\theta_W} \, H_i^0 Z_\mu Z^\mu , \tag{6}$$

where c_i is a measure of the scaling with respect to the SM Higgs coupling. A sum rule for the c_is follows:

$$\mathcal{C} \equiv \sum_j c_j^2 = 4 \frac{\sum_i T_{3i}^4 v_i^2}{\sum_i T_{3i}^2 v_i^2} . \tag{7}$$

Clearly, $\sum_j c_j^2$ increases if representations with larger hypercharge get relatively larger VEVs. On the other hand, the presence of an arbitrary number of doublets (and singlets) does not change $\mathcal{C}$ from its SM value of unity. In fact, $\mathcal{C}$ is bounded from both above and below[2]

$$4\max(T_{3i}^2) \geq \mathcal{C} \geq 4\min(T_{3i}^2) \tag{8}$$

with the extremal values being reached in the event of a particular VEV dominating all others. A clear departure from the Standard Model relation $\mathcal{C} = 1$ would thus indicate the presence of new physics.

Were we interested only in maximizing $|\mathcal{C} - 1|$ while maintaining $\rho = 1$, the result would be given by eq.(8). However, apart from the consequent fine-tuning amongst the VEVs of the higher representations, a further aspect needs to be considered. As we have already remarked, eq. (4) implies that were higher representation to have nonzero VEVs, the magnitude of doublet VEV(s) would necessarily be smaller than the Standard Model value. An immediate consequence is that the Yukawa couplings leading to the fermion mass terms would need to be scaled up. Restricting ourselves to the simplest (and *conservative*) case of only one Higgs doublet ($\Phi_{1/2}$) being responsible for the fermion mass generation, the Yukawa couplings are now given by

$$y_f = y_f^{\rm SM} \frac{v_{\rm SM}}{v_{1/2}} \ . \tag{9}$$

Future experiments, both at the LHC [7] and at a linear collider [12] will serve to measure the Yukawa couplings for the top- and bottom-quarks as well as the τ-lepton, at least for intermediate values of the Higgs mass[3]. *Assuming that the theory contains only one doublet*, this information could then be used to set stringent limits on parameters such as $v_{1/2}/v_{\rm SM}$. In other words, such measurements would immediately alert us about the presence of a nonminimal Higgs sector. In fact, an accurate measurement of more than one Yukawa coupling could also shed light on the presence of more than one doublet. We will later consider the explicit example of a triplet Higgs model.

Is the analysis of this paper redundant then? Note that if the Yukawa couplings are found to be very close to the SM ones, it is extremely unlikely that a renormalizable four-dimensional theory could accommodate a value of $\mathcal{C}$ significantly different from unity. Identification of a second neutral scalar and a measurement of an anomalous value of $\mathcal{C}$ would then immediately point towards either a radion or a SM singlet with higher-dimensional couplings with the SM fields. On the other hand, if the Yukawas are found to be quite different from their SM values, the only unambiguous conclusion that we may draw from such measurements is the presence of *some* nontrivial structure of the EWSB sector. An analysis as proposed in this paper could, once again, prove very useful in our quest to unravel the mystery of this sector.

Concentrating on the top Yukawa, a larger value at low energies would imply a faster growth as one evolves it to larger energies. The one-loop[4] renormalization group equation for the top

[2]Singlet VEVs, of course, are not being considered here.

[3]Note that the analyses of these papers implicitly assume that the Higgs couplings, whether gauge or Yukawa, are relatively close to the SM ones. A breakdown of this assumption could lead to significant changes in the results obtained.

[4]While higher loop results are available, it is sufficient, for our purposes, to consider only the lowest order expression.

Yukawa reads

$$2\pi \frac{\mathrm{d}\, y_t^2}{\mathrm{d}\, \ln\mu} = -\left[8\alpha_3 + \frac{9}{4}\alpha_2 + \frac{17}{12}\alpha_1\right] y_t^2 + \frac{9}{8\pi} y_t^4 \ , \tag{10}$$

where μ is the scale and α_i represent the (scale dependent) gauge couplings within the Standard Model. Clearly, if $y_t(m_t)$ were to be larger, the y_t^4 piece on the right hand side would tend to dominate, thereby driving up the Yukawa coupling as the scale increases. The scale of nonperturbativity, $\mu_{\rm nonpert}$, may be defined through

$$y_t(\mu_{\rm nonpert}) = \sqrt{4\pi} \ . \tag{11}$$

In fig.1, we exhibit the functional dependence of $\mu_{\rm nonpert}$ on the value of the top-Yukawa at the pole, namely $y_t(m_t)$. As the figure clearly shows, if we require the theory to be perturbative until very high scales, $y_t(m_t)$ would be required to assume values close to its Standard Model magnitude.

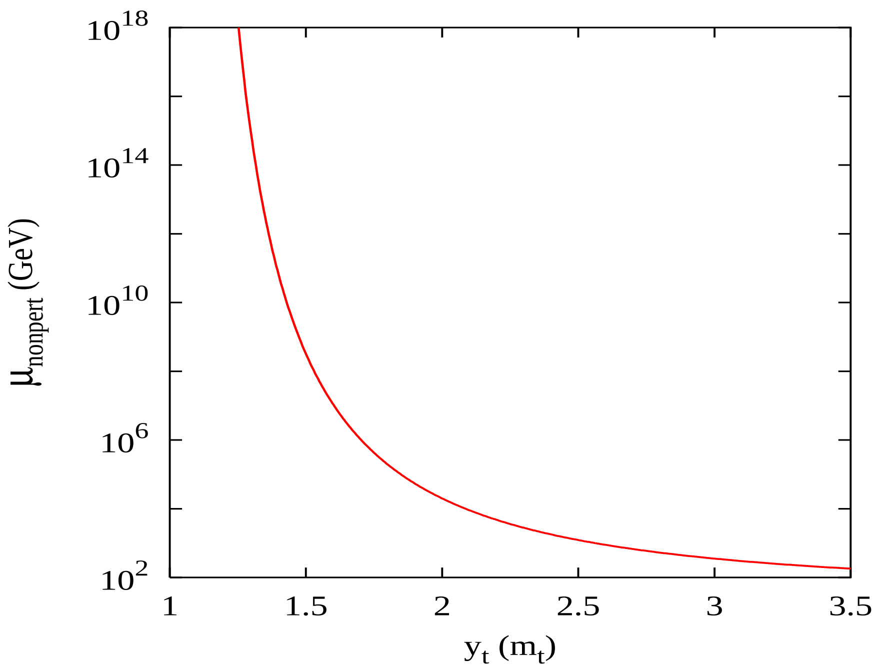

Figure 1: *The scale at which the top-quark Yukawa coupling becomes nonperturbative (see eq.(11)) as a function of its value at the pole.*

Having developed the tools, let us now proceed to investigate, numerically, the extent to which the sum $\mathcal{C}$ may vary in the presence of extra scalars, while satisfying experimental bounds. Rather than complicate matters by including scalars in arbitrary representations, we restrict ourselves to scenarios wherein the Standard Model doublet is accompanied by either $SU(2)_L$ triplets or fourplets or fiveplets. Within each such class, we do not further restrict the value of the hypercharge Y. We could, of course, have considered extra doublets (or singlets), but such inclusion would not have virtually any bearing on our numerical results.

Triplet Higgses.

Apart from the doublet [(2, 1/2)], we now have an arbitrary number of real [(3, 0)] and complex [(3, 1)] triplets. Clearly, a VEV for the former increases ρ (see eq.(5)) from its Standard Model value of unity, while that for the latter serves to reduce it. Thus, experimental consistency stipulates that the presence of a non-zero VEV for one kind must be accompanied by a VEV for at least one of the other kind.

For the sum of the squared couplings, we now have $1 \leq \mathcal{C} \leq 4$ with the upper limit being reached when at least one of the complex triplets (and hence at least one of the real triplets) have a VEV much larger than the doublet. An examination of eq. (7) shows that the presence of a multitude of triplets would not change the upper bound on $\mathcal{C}$ in any way. Rather, for a given scaling of the top Yukawa (i.e. for the VEV of the doublet), the maximal value of $\mathcal{C}$ is reached when the extra contribution to the gauge boson masses accrues from only a single set of triplets. In our quest of identifying the maximal possible deviation from the value of $\mathcal{C}$, we may thus limit ourselves to a study of just one such pair. The results are presented in fig. 2 as a function $y_t(m_t)$. The width to the curve allowed by the experimental uncertainties in the measurement of ρ is too small to be discernible. As expected, for $y_t(m_t) \approx y_t^{\rm SM}(m_t)$ (namely a very small scaling of the doublet VEV), $\mathcal{C} \approx 1$. On the other hand, even for an arbitrarily large $y_t(m_t)$, which corresponds to the bulk of the gauge boson masses arising from the triplet VEVs, $\mathcal{C}$ only reaches its asymptotic value of 4.

Until now, we have entirely desisted from remarking on the fine-tuning problem in the cancellation of the two (or more) triplet contributions to the ρ-parameter. The degree of fine-tuning may be quantified in terms of the maximal fractional deviation, $(\delta v_i/v_i)_{\rm max}$, that a given triplet VEV v_i may suffer without coming in conflict with the experimental results. In fig. 2, we also show the 'bounds' corresponding to a 95% C.L. agreement with the data [13]. As is immediately apparent, if we wish the fine-tuning to be no worse than *per mille* level, we would be confined to $y_t(m_t) \lesssim 0.72$ and hence to $\mathcal{C} \lesssim 1.3$, a value quite close to the SM one. However, if we admit a fine-tuning of $\sim 10^{-4}$, the whole range for $\mathcal{C}$ would be admissible.

A constraint that we have not yet imposed is the requirement of perturbativity. As fig. 1 amply demonstrates, $y_t(m_t) \lesssim 3.5$ for perturbation theory to make sense even at the top-mass scale. This still allows $\mathcal{C} \lesssim 3.5$. On the other hand, if we demand that the theory remain perturbative until close to the GUT scale, we are immediately restricted to $\mathcal{C} \lesssim 2.5$. This still represents a very significant deviation from the Standard Model.

Before delving into the discussion of more complex Higgs sectors, we will briefly mention how the experimentally measured Higgs widths can constrain the Higgs sector. An illustrative example is the triplet Higgs model by Georgi and Machachek (GM) [10]. Couplings of the SM-like Higgs of the model to gauge bosons and fermions are suppressed/enhanced with respect to the SM case by a factor of $v_{1/2}/v_{\rm SM}$. Therefore, measurement of these factors at future experiments can shed light on the physical model. E.g. the partial decay width into a fermion pair can be expressed in terms of the SM width as $\Gamma^{ff}_{GM} = (v_{\rm SM}/v_{1/2})^2\ \Gamma^{ff}_{SM}$. Assuming the ratio of the VEVs is not too different from unity, the signature profile at a collider, and hence the experimental efficiencies, would not change drastically. One can use the experimentally

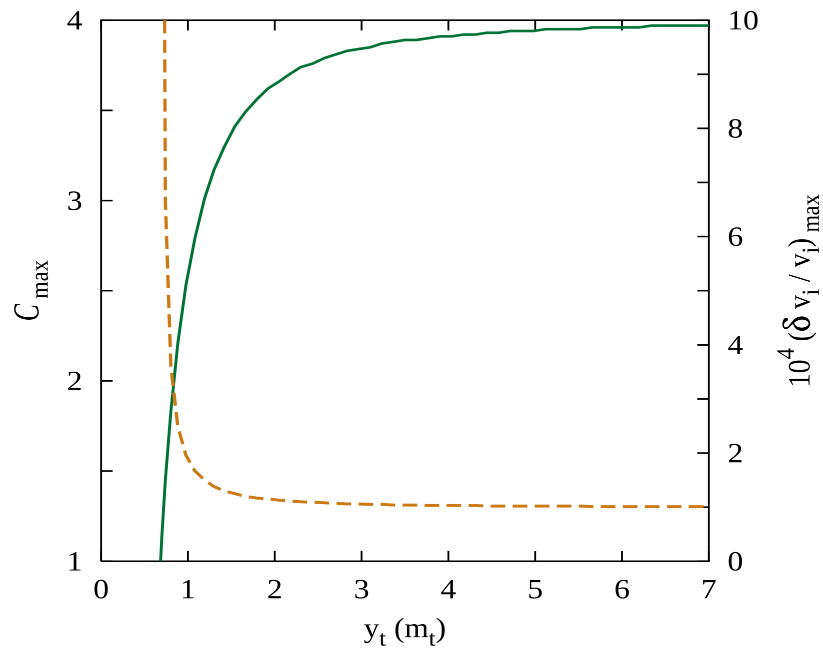

Figure 2: *The solid line represents the quantity $\mathcal{C}$ as a function of the top Yukawa for a model with the standard (one doublet) Higgs sector augmented by one each of real and complex triplets. The tree level constraint $\rho = 1$ has been imposed. For an arbitrary number of triplets satisfying this constraint, $\mathcal{C}$ is restricted to be* below *this curve. The dashed line gives the maximum fractional deviation of a triplet Higgs VEV that is allowed by the data at 95% C.L.*

measured value (Γ^{exp}_{ff}) of the ff width and its error ($\Delta\Gamma^{exp}$) to constrain the ratio of VEVs at a certain confidence level using the inequality

$$\frac{\Gamma^{exp}_{ff} - \alpha\Delta\Gamma^{exp}}{\Gamma^{SM}_{ff}} \lesssim (v_{\rm SM}/v_{1/2})^2 \lesssim \frac{\Gamma^{exp}_{ff} + \alpha\Delta\Gamma^{exp}}{\Gamma^{SM}_{ff}}, \tag{12}$$

where α depends on the confidence level at which one would want to constrain the ratio.

The projected accuracy on $\Gamma(h \to \tau\tau)$ measurement at the LHC is of the order of $10-20\%$ [7]. An e^+e^- linear collider fares much better in this respect. The accuracy [12] of the Higgs width measurement can be as good as 5% instead. From the above expression, eq. (12), it is evident how the experimental accuracy determines the constraint on the ratio.

As the results for 4-plet Higgses are qualitatively very similar to those obtained above, we move straightaway to the 5-plets.

5-plet Higgses.

Apart from the doublet [(2, 1/2)], we now have an arbitrary number of real [$F_0 \equiv (5,0)$] and complex [both $F_1 \equiv (5,1)$ and $F_2 \equiv (5,2)$] 5-plets. Clearly, VEVs for the first two types can only enhance ρ while that for a F_2-type field suppresses it. Thus, to satisfy the experimental constraint in presence of 5-plet VEVs, one needs at least one VEV of the (5, 2) kind.

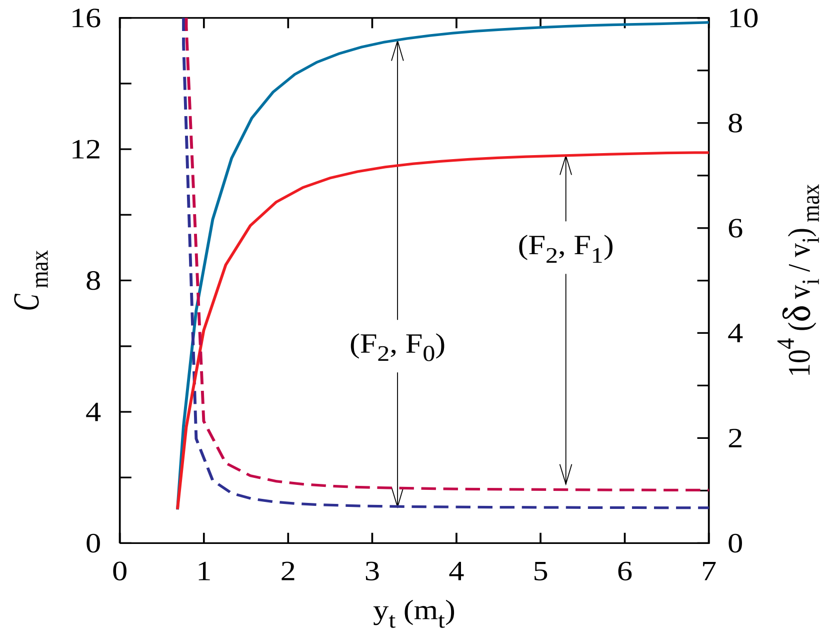

Figure 3: *The quantity $\mathcal{C}$ (solid lines) as a function of the top Yukawa for a model with a doublet Higgs and arbitrary number of real and complex 5-plets and respecting the tree level constraint $\rho = 1$. The two curves correspond to the cases with no F_0 type VEV and no F_1 type respectively. The dashed lines give the maximum fractional deviation of a 5-plet Higgs VEV that is allowed by the data at 95% C.L. If both kinds are present, the corresponding curves would lie in between the two respective sets.*

For the sum of the coupling squares, we now have the inequality

$$1 \leq \mathcal{C} \leq 16 \ , \tag{13}$$

with the upper limit being reached when at least one of the F_2's (and hence at least one of the other triplets) have a VEV much larger than the doublet. Once again, for a given $y_t(m_t)$, the maximal value of $\mathcal{C}$ is reached when a single F_2-type field acquires a large VEV and, for simplicity, we shall illustrate our results under this assumption. There are two interesting limits though. If no F_0 gets a VEV, $\mathcal{C}(y_t)$ follows a particular limit

$$\mathcal{C}(F_2, F_1) = \frac{288\langle(5,2)\rangle^2 + \langle(2,1/2)\rangle^2}{24\langle(5,2)\rangle^2 + \langle(2,1/2)\rangle^2} \ ,$$

with an ensuing maximum of 12 (see fig.3). This is irrespective of the actual numbers of F_2's and F_1's, as along as at least one of each kind exists. In the opposite case (no F_1 gets a VEV), $\mathcal{C}$ moves along the other extreme

$$\mathcal{C}(F_2, F_0) = \frac{256\langle(5,2)\rangle^2 + \langle(2,1/2)\rangle^2}{16\langle(5,2)\rangle^2 + \langle(2,1/2)\rangle^2} \ ,$$

and reaches the absolute maximum of 16. Both these curves are displayed in fig. 3. The results for the mixed case lie somewhere in between these two extremes depending on the relative magnitudes of the VEVs. Also displayed in the same figure are the fine-tuning curves corresponding to either of these branches. Once again, if we limit ourselves to a fine-tuning no worse than 10^{-3}, we are limited to $\mathcal{C} \lesssim 2$.

The arguments concerning perturbativity remain essentially the same as in the case of triplets.

3 ZZH coupling in higher dimensional models

Theories with TeV-scale gravity [14, 15] have recently been proposed as an alternative to supersymmetry as the solution for the hierarchy problem. Such models, perforce, have to be defined in dimensions larger than four. Since the results of the present day gravity experiments demonstrate excellent agreement with Newtonian gravity down to a distance scale about 0.1 mm, the compactification radius for the extra dimensions must be smaller than this scale. In the simplest examples of such models, all the SM fields are assumed to be confined to 4-dimensional hypersurfaces or branes.

Two different explanations for the perceived hierarchy between the Planck-scale and the fundamental ($\sim$ TeV) scale exist. In the first class of models [14], the metric in the higher dimensional theory is assumed to be a factorizable one, and the large volume of the compactified dimensions leads to the aforementioned hierarchy. Such models are characterized by very closely spaced Kaluza-Klein tower(s) of the graviton.

The model proposed by Randall and Sundrum (RS) [15], on the other hand, is based on the premise that the 5-dimensional theory is described by AdS_5 geometry. With the 4-dimensional part of the metric suffering an exponential warp depending on the fifth co-ordinate, the perceived scale of gravity on two branes separated in the fifth direction would naturally differ enormously. The lowest Kaluza-Klein excitation of the graviton is now at the TeV scale. Interestingly, the model also contains an extra scalar, the *radion*, which may be lighter than the graviton excitations and may also mix with the SM Higgs thereby changing the relevant phenomenology. It is this class of models that we shall concentrate on in this section.

While the models with TeV scale gravity were originally deemed as alternatives to low energy supersymmetry, it was subsequently realized that the two ideas could well be linked to each other. For one, the very stabilization of the brane-world picture may need underlying supersymmetry, especially if they are to represent vacua of superstring theories. At a more phenomenogical level, theories with extra dimensions seem to provide a natural way of communicating supersymmetry breaking from a hidden sector onto our world [16, 17].

We will, hence, consider both the non-supersymmetric and supersymmetric versions of the RS-model.

3.1 Non-supersymmetric RS-model.

In the model proposed in ref. [15], possibly the lowest mass excitation from the gravitational sector is a scalar, the radion. This corresponds to the 4-dimensional manifestation of g_{55} of the full 5-dimensional metric and can also be visualised as the distance between the two branes situated at the two fixed points of the orbifold on which the extra dimension is compactified.

As is obvious, the radion needs to be stabilized for the effective 4-dimensional Planck mass to *remain* exactly what we perceive it to be. In Ref. [18], Goldberger and Wise demonstrated that the inclusion of a bulk scalar, coupling minimally, to gravity can lead to an effective potential for the radion and hence to its stabilization. The mass of the radion turns out to be lower than that for the lowest $J = 2$ Kaluza-Klein excitation, and hence the radion is likely to offer the first signature of the brane world picture.

It can be shown from very general considerations that the radion couples to the SM fields via the trace of the energy momentum tensor. In addition, the effective action may contain terms leading to curvature-Higgs mixing [19, 20]. At the two-derivative level, the coupling of the gravity and the Higgs H — confined to the visible brane — may be parametrized as

$$S = -\xi \int d^4x \sqrt{-g_{vis}}\, R(g_{vis})\, H^\dagger H, \tag{14}$$

where the Ricci scalar $R(g_{vis})$ is obtained from the induced four dimensional metric on the visible brane. Clearly, the above term induces a kinetic mixing between the Higgs and the radion. Additional mixing is introduced by the fact that both the neutral component of the Higgs (h) as well as the radion (ϕ) acquire VEVs:

$$\langle H \rangle = v, \quad \langle \phi \rangle = \Lambda, \quad \text{with} \quad \gamma \equiv \frac{v}{\Lambda} \ . \tag{15}$$

To obtain fields with canonical quantization rules, it is necessary to effect field redefinitions:

$$\begin{pmatrix} \phi \\ H \end{pmatrix} \to \begin{pmatrix} \phi' \\ H' \end{pmatrix} \equiv Z_R\, \mathcal{M}^{-1} \begin{pmatrix} \phi \\ H \end{pmatrix} \ , \qquad \mathcal{M} = \begin{pmatrix} \cos\theta & -\sin\theta \\ Z_R \sin\theta - 6\xi\gamma\cos\theta & Z_R\cos\theta + 6\xi\gamma\sin\theta \end{pmatrix} \tag{16}$$

where

$$Z_R^2 = 1 - 6\xi\gamma^2(1+6\xi) \quad \text{and} \quad \tan 2\theta = \frac{12\xi\gamma Z_R m_h^2}{m_H^2(Z_R^2 - 36\xi^2\gamma^2) - m_\phi^2}. \tag{17}$$

In the limit $\xi \to 0$, we recover back the SM Higgs from $H^{'}$. Requiring that the kinetic terms for the physical fields H' and ϕ' be positive, restricts us to [20]

$$\frac{-1}{12}\left(1 + \sqrt{1 + 4/\gamma^2}\right) \le \xi \le \frac{1}{12}\left(\sqrt{1+4/\gamma^2} - 1\right). \tag{18}$$

For $\Lambda = 1$ TeV, this translates to $-0.75 < \xi < 0.59$, while for $\Lambda = 10$ TeV, the constraint is much weaker ($-6.7 < \xi < 6.6$).

The couplings of the physical scalars to a generic gauge boson V can be expressed as [21]

$$\mathcal{L} = -\frac{M_V^2}{v}\, V_\mu V^\mu \ \left[(\mathcal{M}_{22} + \gamma\mathcal{M}_{12})\ H + (\gamma\mathcal{M}_{11} + \mathcal{M}_{12})\ \phi\right] \ . \tag{19}$$

The parenthetical coefficients directly give the strength of the corresponding interaction when compared to the case with no mixing[5], and hence correspond to the quantities $c_{H'}$ and $c_{\phi'}$ (see eq. (6)). An important observation needs to be made here. Even in the absence of any Higgs-curvature mixing ($\xi = 0$), the radion couples to the gauge bosons *without* affecting their coupling

[5]Note that we neglect here the small correction due to the conformal anomaly.

to the Higgs. Since the latter coupling is responsible for unitarizing the gauge boson scattering amplitudes ($V_iV_j \to V_kV_l$) and since the radion-exchange contribution to these amplitudes are quite similar to the Higgs-exchange ones, it is obvious that the presence of the radion would destroy the partial wave unitarity for such amplitudes [22, 23]. This is quite unlike the case of the multi-Higgs models in renormalizable 4-dimensional theories. Quantitatively, the unitarity constraint has only a weak dependence on γ [22] and can be approximated by $|\xi| \lesssim 2.7$.

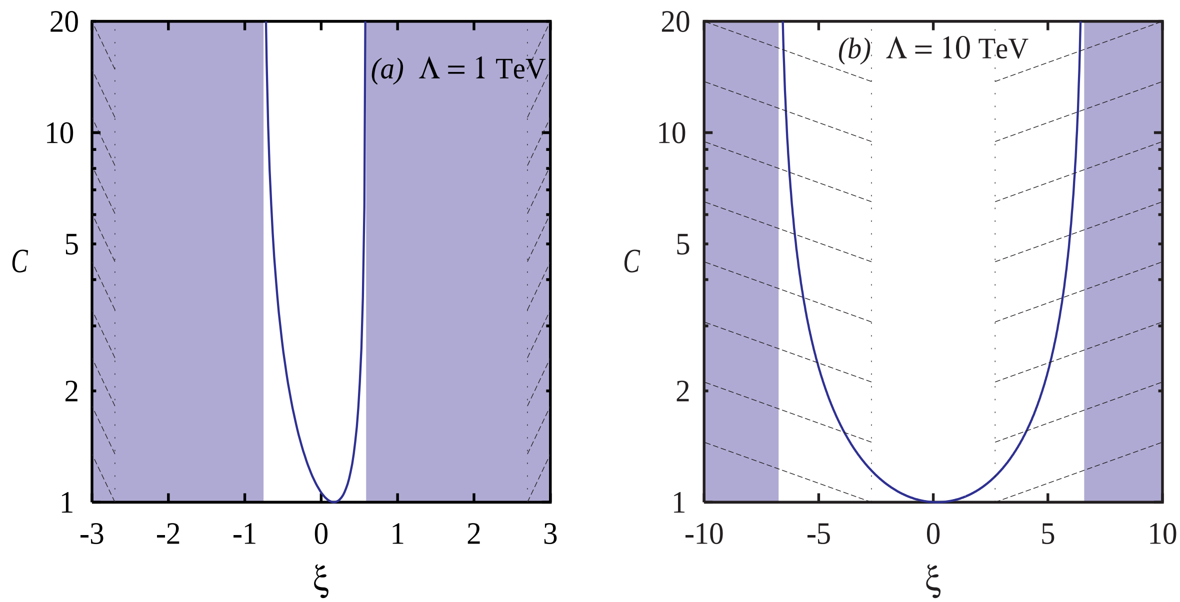

Figure 4: *The parameter $\mathcal{C}$ as a function of radion-Higgs mixing parameter ξ for two different values of Λ, the radion VEV. The shaded area in each plot is ruled out by the requirement that the kinetic energy be positive definite, while the hatched area would lead to a loss of partial wave unitarity.*

Reverting back to our measure for distinguishability, we now have

$$\mathcal{C} = (\mathcal{M}_{22} + \gamma \mathcal{M}_{12})^2 + (\gamma \mathcal{M}_{11} + \mathcal{M}_{12})^2 = 1 + \frac{\gamma^2}{Z_R^2} (1 - 6\xi)^2 \ , \tag{20}$$

which clearly indicates a deviation from the SM result of $\mathcal{C} = 1$ *even in the absence of curvature-Higgs mixing.* However, the extent of deviation, in such cases, is very small and is unlikely to be detectable. The presence of a mixing term, though, can enhance the effect manifold, as is borne out by fig. 4. That the effect should decrease with increasing Λ, the radion VEV, is expected. The figure also amply demonstrates the relative importances of the constraints from the positivity of the kinetic energy and from perturbative unitarity. At this stage, it should be noted that the radion-gauge boson coupling is the driving force behind a large $\mathcal{C}$. In other words, it is the $\phi' ZZ$ coupling (ϕ' is the radion-dominated scalar) that grows very fast with increasing ξ whereas the $H'ZZ$ coupling never grows beyond its SM counterpart.

From an experimental point of view, it is worth asking how much of the parameter space one could possibly distinguish from the Standard Model. The answer, of course, would be a

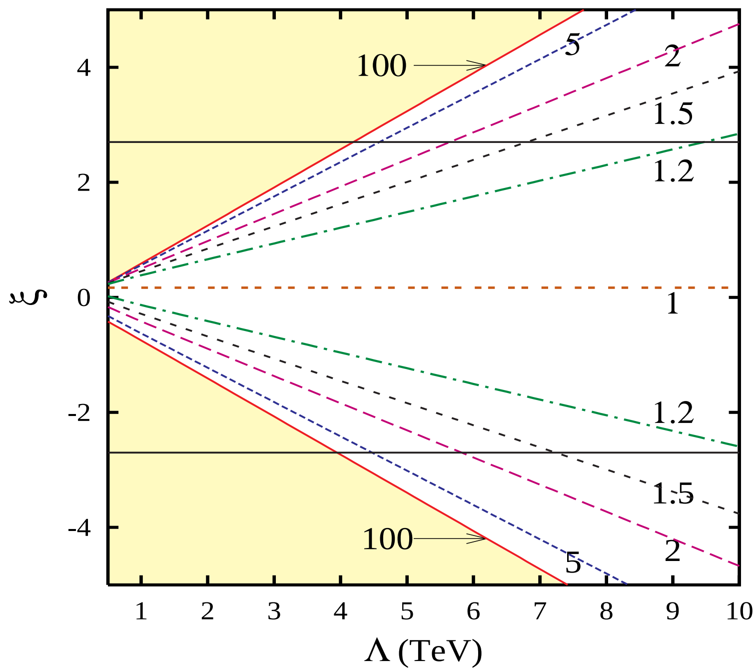

Figure 5: *Contours of constant $\mathcal{C}$ in the Λ-ξ plane for the nonsupersymmetric RS scenario. The shaded regions correspond to the unphysical domain of negative kinetic energy. Requiring that gauge-boson scattering amplitudes respect perturbative unitarity would restrict one to the region* between *the two horizontal solid lines.*

function of the sensitivity of the experiment under consideration. An indication, though, may be obtained from fig. 5, wherein we have plotted iso-$\mathcal{C}$ contours in the Λ-ξ plane. The contours are almost linear, with deviations from linearity becoming apparent only for $\Lambda \sim v$. The contour corresponding to absolute indistinguishability from the SM ($\mathcal{C} = 1$) is but the straight line $\xi = 1/6$. As one moves away from $\mathcal{C} = 1$, the contour splits up into two disjoint curves. For large $\mathcal{C}$, the curves asymptotically reach the boundary separating the unphysical domain ($Z = 0$). It is quite apparent that our measure ($\mathcal{C}$) is unlikely to differentiate a RS theory with large Λ unless it is also accompanied by a large Higgs-curvature mixing. For radion VEVs of a few TeV, on the other hand, marked deviations from the SM expectation would be obtainable even for relatively small mixings.

3.2 Supersymmetric RS-model.

A particularly important issue for Randall-Sundrum-type models relates to the (phenomenologically needed) stabilization of the radion. Several schemes have been proposed, especially in the context of the supersymmetric versions [24, 16, 17]. Interestingly, in such proposals, supersymmetry breaking and radius stabilization are intimately linked to each other.

As a concrete example of such scenarios, we shall consider in this work the low energy model discussed in ref. [25]. The standard SUGRA particle content needs to be supplemented by a complex scalar field[6] $T = k\pi(r + ib\sqrt{2/3})$ [16]. In addition, one needs matter in the bulk in order to ensure supersymmetry breaking [24]. This matter is represented by a universal hypermultiplet S. In the model of ref.[25], the warp factor is given by

$$e^{-\langle T+T^\dagger\rangle} = \frac{\Lambda^2}{3M_P^2}, \tag{21}$$

where $\Lambda = \mathcal{O}(\text{TeV})$ and $T = \langle T\rangle + t/\Lambda$. Here t is the radion superfield, which will mix with the Higgs fields through the kinetic and mass terms. The term in the Lagrangian relevant to the matter chiral superfields on the visible brane is given by

$$\mathcal{L} = \frac{1}{2} G_{ij} D_\mu \phi_i D^\mu \phi_j, \tag{22}$$

where G_{ij} is obtained from the effective Kähler potential in the usual way:

$$\begin{aligned} G_{ij} &= \frac{\partial^2 K_{eff}}{\partial\phi_i \partial\phi_j} , \\ K_{eff} &= \Lambda^2 \exp\left\{ -\frac{t+t^*}{\Lambda} + \frac{1}{\Lambda^2}\Sigma_{eff}(\{\phi_i\}) \right\} . \end{aligned} \tag{23}$$

with $\Sigma_{eff}(\{\phi_i\}) = \Sigma_i |\phi_i|^2 + \lambda(H_1 \cdot H_2 + h.c.)$.

Since the field t is a gauge singlet, the kinetic term does not lead to a tZZ coupling. However, such a term does arise from the trace of the energy-momentum tensor and is given by

$$L_{tZZ} = -tZ_\mu Z^\mu \frac{m_Z^2}{\Lambda}. \tag{24}$$

For the MSSM scalars H_1 and H_2, on the other hand, we now have

$$L_{HZZ} = \left(1 + (\lambda+1)(\lambda+3)\frac{v^2}{4\Lambda^2}\right) \frac{g}{4\cos\theta_W}(v_1 H_1^0 + v_2 H_2^0) Z_\mu Z^\mu, \tag{25}$$

where v_i are the VEVs of H_i^0 and $v^2 = v_1^2 + v_2^2$. The mass of the Z boson is then

$$m_Z = \frac{gv}{2\cos\theta_W} \left[1 + (\lambda+1)(\lambda+3)\frac{v^2}{4\Lambda^2} \right]^{1/2} . \tag{26}$$

An interesting feature of this model relates to the ratio of the VEVs for H_i^0, namely $\tan\beta \equiv v_2/v_1$. The potential is minimized for $\tan\beta = 1$ [25]. This is quite unlike the case of the MSSM wherein $\tan\beta$ is a free parameter, and current experimental data strongly disfavour $\tan\beta \sim 1$. This difference can be traced back to the additional contributions that the Higgs mass receives from its coupling to the radion (vide eq.23). A consequence of $\tan\beta = 1$ is that, of the two orthogonal combinations

$$\phi^0 = \frac{1}{\sqrt{2}}(H_1^0 + H_2^0) \quad \text{and} \quad H^0 = \frac{1}{\sqrt{2}}(H_1^0 - H_2^0) , \tag{27}$$

[6] Here $\langle r\rangle$ is the radius of compact S^1 and $\langle b\rangle$ the Aharanov-Bohm phase of the graviphoton around the S^1.

ϕ^0 alone contributes to the Z mass, while the coupling of the H^0 to Z vanishes identically. Note that the redefinition of eq.(27) does not entirely diagonalize the kinetic terms. To achieve that goal, one has to effect a further field redefinition [25] akin to that performed in Section 3.1 and involving all three of $\{H_1^0, H_2^0, t\}$. On doing this, we obtain, for the sum of the squares of the scalar couplings to the Z (and normalized to the SM value),

$$\mathcal{C} = \left[1 + \frac{v^2}{2\Lambda^2}(1+\lambda)\left\{\frac{1}{\sqrt{2}} - \frac{\lambda+5}{8}\right\}\right]^2 + \frac{v^2}{\Lambda^2}\left[1 + \frac{(1+\lambda)}{2\sqrt{2}} - \frac{v^2}{16\Lambda^2}(1+\lambda)(1-3\lambda)\right]^2 . \quad (28)$$

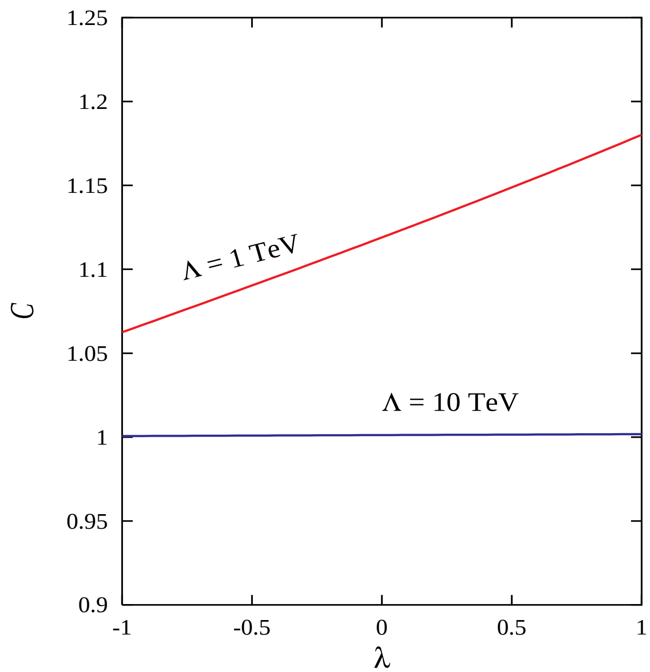

Figure 6: *The parameter $\mathcal{C}$ as a function of parameter λ. The two lines correspond to $\Lambda = 1$ TeV and $\Lambda = 10$ TeV. respectively*

In fig.6, we present $\mathcal{C}$ as a function of the Higgs mixing parameter λ. It is evident from eq. (28), that $\mathcal{C}$ grows with λ. One sees easily that, for a large Λ, the deviation from the Standard Model value of unity is negligible. However, for a relatively small Λ, the effect might be noticeable.

4 Discussion and conclusions

To summarize, we have studied the ZZH coupling in models with different types of scalar sectors, but with the Standard Model gauge group. We establish that the sum of the squares of the couplings of the scalars to the Z can be an efficient discriminator. For scenarios with non-zero VEVs for Higgses in higher dimensional representations of $SU(2)_L$, this quantity can differ substantially from the SM value of unity. However, if we demand either naturalness in the value for the ρ-parameter or demand that the top Yukawa coupling stays perturbative until GUT-scale, this parameter is restricted to values close to one.

In higher dimensional models with warped geometry, on the other hand, the effect of Higgs-curvature mixing on this parameter may be quite significant. This remains true even after the constraints from perturbative unitarity are imposed. Thus, such a model would be clearly distinguishable from a "natural" multi-Higgs model provided the radion VEV is less than a few TeVs. Imposition of supersymmetry on such models, however, renders this measure largely ineffective. In either case, the radion, being a gauge singlet, does not contribute to fermion masses and consequently no constraints emerge from this sector.

Before we conclude, we would like to make some brief comments about the ZZH couplings in the recently proposed model of 'the littlest Higgs' [26]. The local gauge symmetry, at high energies, is enlarged to $[SU(2) \otimes U(1)]^2$. This is broken down to the familiar $SU(2)_L \otimes U(1)_Y$ at a scale $\Lambda \sim \mathcal{O}(\text{TeV})$. The scalar potential at low energy is generated by mechanism very similar to that of Coleman and Weinberg [27] and the details have been presented in ref.[28]. The physical spectrum includes two neutral CP even, one neutral CP odd, one charged and one doubly-charged scalars. In contrast to the RS or SUSY-RS models, the extra scalars are not gauge singlets any more and all the fermions, in general, can have extra contributions to their masses of $\mathcal{O}(v/\Lambda)$. If we concentrate on the couplings of the two neutral CP-even Higgses to the pair of Zs, and calculate the quantity $\mathcal{C}$ as we have done earlier, the expression looks qualitatively very similar to eq. (28). In other words, as we increase the scale Λ, the coupling of one of these states to a pair of Z tends to that of the SM case while the other decouples from SM. The results are similar to those for the SUSY-RS case.

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